Ch 10 and 11 HW

10-1 Show that for given E, smaller rest mass leads to smaller wavelength.

$$E^2 = p^2c^2 + E_0^2$$

$$= \left(\frac{h}{\lambda}\right)^2 c^2 + (m_0c^2)^2$$

solve $\lambda^2 = \dfrac{h^2}{E^2/c^2 - \dfrac{(m_0c^2)^2}{c^2}}$

$E>m_oc^2$ So, smaller mass increases the denominator for a fixed Energy E. The wavelength reduces.

10-5 Given a neutron, 1.675E-27 kg and wavelength of 100pm, how fast?

$$p = \gamma m_0 v = \frac{h}{\lambda}$$

$$\gamma v = \frac{h}{m_0\lambda}$$

square

$$\frac{v^2}{1 - v^2/c^2} = \frac{h^2}{m_0^2\lambda^2}$$

Algebra - solve v

$$v = \frac{(h/m_0\lambda)}{\sqrt{1 + (h/m_0\lambda)^2 \left(\frac{1}{c}\right)^2}}$$

$= 3956$ m/s

Slow, so we could have just ignored gamma. Could have checked energy to start.

10-6 Express wavelength =h/p in terms of KE for relativistic various cases (a,b,c, d)

$$E^2 = p^2c^2 + E_0^2$$

a) photons so $E_0 \to 0$ $E \to KE$

$$\lambda = \frac{h}{p} = \frac{hc}{KE}$$ ✓

b) very rel. so $E \gg E_0$

Drop E_0 $E \sim KE$ same approx result

$$\lambda \approx \frac{hc}{E} \approx \frac{hc}{KE}$$

So it looks like photons are as relativistic as things can get.

c) General $p = \frac{1}{c}\sqrt{E^2 - E_0^2}$

$$= \frac{1}{c}\sqrt{(E-E_0)(E+E_0)}$$

$$= \frac{1}{c}\sqrt{KE(KE + 2E_0)}$$

$$\lambda = \frac{hc}{\sqrt{KE(KE + 2E_0)}}$$

We have been asked to write result in terms of KE and constants. E_0 is a constant.

d) Classical — so $KE \ll E_0$

$$\lambda = \underbrace{\frac{hc}{\sqrt{KE(2E_0)}}}_{\text{result}} \approx \frac{hc}{\sqrt{\frac{1}{2}m_0v^2(2m_0c^2)}} = \frac{h}{m_0v}$$

10-13

Calculate phase velocity of relativistic particle with rest mass greater than zero.

Phase velocity refers to speed of a wave with given frequency and wavelength---many ways to say that.

$$y = A \sin(kx - \omega t)$$

$$V_{phase} = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{h\nu}{h/\lambda} = \frac{E}{p}$$

$$= \frac{\sqrt{p^2c^2 + E_0^2}}{p} = c\frac{\sqrt{p^2 + m_0^2}}{p}$$

So, for $m_0>0$, the phase velocity is greater than c. Note this does not mean any"thing" travels faster than light. Phase carries no energy, no information by itself.

10-14 Show that as group velocity tends to zero, the phase velocity becomes infinite.

We had

$$V_g = \frac{dE}{dp} = \frac{pc^2}{E} \qquad \begin{matrix} d0 \\ lf \end{matrix} \qquad E^2 = p^2c^2 + E_0^2$$

$$= \frac{c^2}{V_{phase}}$$

Clearly the phase velocity goes to infinity as group velocity goes to zero---except for zero rest mass particles where p=E/c. So a particle with zero velocity (or zero momentum) is "everywhere"---

11-1

What is the acceleration of an alpha particle at /near surface of gold nucleus.

$$m_\alpha = 6.7 \times 10^{-27}\ Kg$$

From example 11.1 we Have the electric field "at" the surface of the nucleus.

$$E = 2.0 \times 10^{21}\ N/C$$

$$a = \frac{qE}{m} = 9.6 \times 10^{28}\ m/s^2$$

That is a lot of "g's".   There is no limit in relativity on acceleration.

11-6

Show that E(n)=-KE(n) in the Bohr model, and that KE(n)<<rest energy of the electron.

$$PE = \frac{-Ze^2}{4\pi\epsilon_0 r} \qquad KE = \frac{Ze^2}{8\pi\epsilon_0 r}$$

so $\qquad\qquad = -\frac{1}{2} PE$

$$E = PE + KE$$
$$= PE + \left(-\frac{1}{2}PE\right)$$
$$= +\frac{1}{2}PE$$
$$= -KE$$

For given n $\quad |E_n|_{max} = 13.6\ eV \ll 0.511\ MeV$

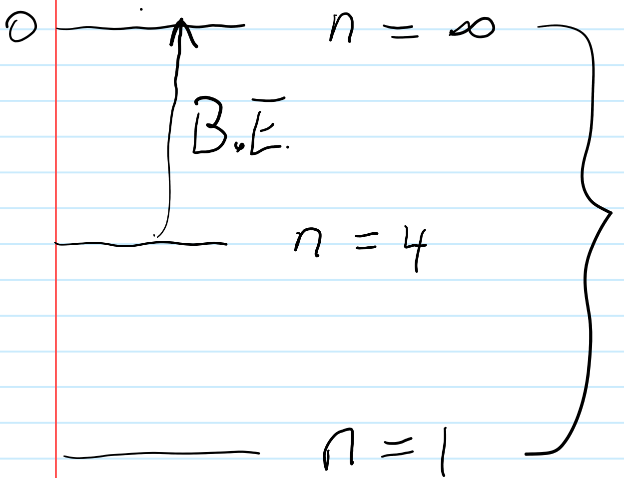
11.7 $\quad$ orbit for $n = 120$

52.9 pm

52.9

$$r_{120} = (120)^2 r_0$$
$$= 762\ nm - \underline{\underline{b19}}$$

11-9 What is binding energy for 3rd excited state (n=4 is 3rd excited state).



$$E_4 = \frac{-13.6\,eV}{4^2}$$

$$= -0.85\,eV$$

excitation from $n=1$ to $n=4$

$$= -(0.85) - (-13.6)$$

$$= 12.75\ eV$$

$$B.E. = 0.85\ eV$$

11-16 If the average lifetime of a state in Hydrogen (excited) is ~10E-8 s, then what is energy width of that state and how many revolutions for orbit--in that time.

$$\Delta E = \frac{h}{\Delta t} = \frac{6.626 \times 10^{-34}\ J\ s}{10^{-8}\ s}$$

$$= 6.626 \times 10^{-26}\ J$$

$$= 4.14 \times 10^{-7}\ eV$$

Most states are near 13.6eV above ground state. The first excited state is 10.2 eV above ground! The energy width is small until one reaches a high n state .....when the states get close together, they may overlap (n+1) and n state.

b)

$$v = \frac{nh}{m(2\pi r_n)} = \frac{nh}{m(2\pi)(r_1 n^2)} = \frac{c}{137\ n}$$

$$\#\ rev = \frac{v\,\Delta t}{2\pi r_n} = \frac{c}{137\ n}\left(\frac{10^{-8}\ s}{2\pi\ n^2 r_1}\right)$$

$$\# rev = \frac{1}{n^3} \frac{c}{137\ 2\pi} \frac{10^{-8}\ s/life}{52.9 \times 10^{-12}\ m}$$

$$= \frac{1}{n^3}\ 6.59 \times 10^{7}\ rev/lifetime$$

High states go slow.